Mathematics Study Guide Narrative

Studying Mathematics

To study math, you must do math. It is not a spectator sport — where one should hope to gain substantial progress by merely looking or reading. To effectively and efficiently learn mathematics you need to roll up your sleeves, get out a pencil and paper and work through problems. Just remember that while some math is easy and straightforward, a lot of it is tricky. Don' t be afraid to make mistakes. If you know the basics, the rest should fall into place with a little patience.

Problem Solving

When studying mathematics, you need to know some basic mathematical terms. You also need to be familiar with some fundamental mathematical concepts. You're probably familiar with most of what we're going to cover here, but it's important to be comfortable with these terms and concepts. Make sure you understand the fundamentals before moving on to problem solving! Very often, what looks like complicated math can turn out to be simple math if you can learn to use these fundamental math ideas well.

Once you know the necessary vocabulary and skills you are ready to begin solving problems. There are many types of problems and mathematicians have figured out many strategies to help us solve those problems. The six strategies we'll look at are:

1. **Guess and check**
2. **Make a table**
3. **Look for a pattern**
4. **Use a model**
5. **Simplify**
6. **Eliminate**

Let’s look at these one at a time

1. Guess and Check:

To use this method you simply make an educated guess at the solution and the check the guess against the conditions of the problems. Here is an example:

Problem 1.1: Find two numbers whose sum is 118 and difference is 36.

If you don't know the answer, begin by focusing on the first part, choosing 2 numbers that add up to 118. For example, 100 and 18. When we check the difference we find it to be 100-18 = 82. We learn from this *example* that the difference is too big. So in our next example, we'll pick numbers that are closer to each other, like 90 and 28.

Eventually, we'll try the numbers 77 and 41, which are the numbers we're looking for. Since 77+41=118, and 77-41=36, the problem is now solved.

Try these:

Problem 1.2: The sum of 2 numbers is 57 and the difference is 3. What are the 2 numbers?

Problem 1.3: To encourage Maria to do her math problems her father promised to pay her 25¢ for every problem she got right, but would take away 15¢ for every problem she missed. After working through 20 problems, Maria was paid 20¢ by her father. How many did she get right?

2. Make a Table

Quite often you can solve a difficult word problem by rewriting it using mathematical tools, like tables or lists of numbers. This approach is often effectively used for problems that ask for a probability or chance. It is also useful when used in conjunction with Guess and Check to organize your guesses. Let's look at an example.

Problem 2.1: How many 3 digit numbers can you make with the digits 4, 5, and 6 using each digit only once.

Make a table or list:

4 5 6	4 6 5	5 4 6	5 6 4	6 4 5	6 5 4

Solution: 6.

Notice in the example that the table helps us to work by letting us see all the possibilities when making the list. We begin by writing all numbers that begin with a 4, then all numbers that begin with a 5, and finally all numbers that begin with a 6.

Problem 2.2: Sam is seven years older than Maria. When will Sam be twice as old as Maria?

Sam	8	9	10	11	12	13	14
Maria	1	2	3	4	5	6	7

Solution: When Maria is 7, Sam is 14.

Try these:

Problem 2.3: How many 4 digit numbers can you make with the digits 5 ,6, 7, 8 using each digit only once?

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Problem 2.4: The 80 chairs in the auditorium are arranged in rows with the same number of chairs in each row. When is the number of chairs per row exactly 5 times the number of rows?

rows	1	2							
chairs	80	40							

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3. Look for a Pattern

Patterns are common in mathematics and often unlock the secrets to relationships. In fact a famous mathematician, G.H. Hardy, once said " A mathematician, like a painter or poet, is a maker of patterns." Let's begin by completing some numeric patterns (use the last term to check your pattern)

1) 1, 4, 7, 10,_____, _____, _____, _____, 25
2) 61, 57, 53, _____, _____, _____, _____, 33
3) 720, 360, 180, _____, _____, _____, 11.25

You can use patterns to solve problems. Most patterns can be translated into a rule that will allow you to work similar problems more quickly. Look for the rule or simply expand the sequence using the pattern. Here is an example:

Problem 3.1: An empty CTA bus is picking up passengers at the following rate. One passenger jumped on at the first stop, three jumped on at the second stop, 5 at the third stop, seven at the fourth, and so on. How many passengers got on the train at the 15th stop?

stop	1	2	3	4	5	6	7	8
# passengers	1	3	5	7	9			

.

You may notice there is a pattern here that can be described by a rule. If we multiply a stop number by two, and then subtract one, we'll get the number of passengers getting on at that stop. We could write this as: 2(stop) - 1=passengers. Having found this rule, we can jump directly to 15 and plug it directly into this equation, giving us an answer of 29. We could also get this answer by completing the pattern all the way up to the answer.

Problem 3.2: Suppose you were offered a job and the employer said she would pay your salary as follows: one penny the first day, 2 pennies the second day, 4 pennies the third day, 8 pennies the fourth day, and so on for a month. How much would you make on the 13th day?

day	1	2	3	4	5	6	7	8	9
pennies	1	2	4	8					.

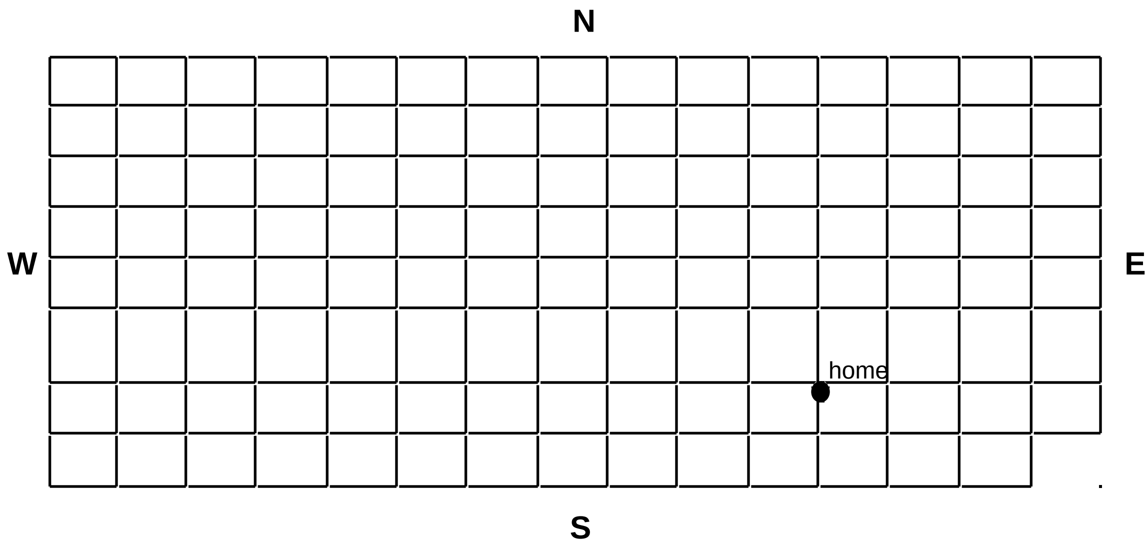
You will be amazed at how much you get paid on the 30th day!

4. Use a Model (or make a picture)

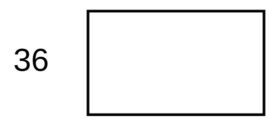
The old saying "a picture is worth a thousand words" is very true in mathematics. A model or picture is often the most effective tool for discovering and explaining mathematical concepts. Consider this example:

Problem 4.1: Suppose you begin walking from home and go 3 miles north, then 3 miles west, then 2 miles south, then 6 miles west, then 1 mile south, then 4 miles east. How far are you from home?

If you trace the journey on the graph below, you'll see that you'll end up 5 miles from home.



Problem 4.2: The length of a tennis court is twice a long as the width. If the court is 36 feet wide, what is the perimeter? To solve this, draw a picture and label the sides of the rectangle.



5. Simplify Strategy

Simplifying a problem involves changing its form so the problem is more easily understood. This in turn usually involves dividing a problem into a series of smaller problems that are more manageable. This strategy is often used in conjunction with one of the other strategies.

Problem 5.1: A palindrome is something that reads the same forward as backward: for example, the number 747. How many palindromes are there between 1 and 500?

An effective strategy would be to simplify the problem by looking at the cases of the number of single or 1 digit palindromes, 2 digit palindromes and 3 digit palindromes. All 1 digit numbers are palindromes. There are nine of them. Two digit palindromes take forms like 11, 22, 33, etc. By extending this pattern, we find out that there are nine of these as well. Three digit palindromes take the form of

101, 111, 121, 131,...
202, 212, 222, 232,...
303, 313, 323, 333,...
404

Extending the patterns, we find that the table of 3 digit palindromes would be 10 rows of 4 numbers = 40.
total = 9 + 9 + 40 = **58**

Problem 5.2: How many 1, 2, and 3 letter palindromes can be made using the alphabet?

This problem is a bit trickier than the first one because letters don't have sequential values like numbers. But if you think in terms of *possible combinations* of letters, this problem begins to fall into place. Since we know that we're only looking for combinations that are palindromes, we can eliminate combinations like "ab" (two letter combination) and "sce" (three letter combination). An example of a two letter palindrome is "gg." An example of a three letter palindrome is "ozo." Good luck!

6. Elimination Strategy

This is a strategy used by people every day. If you can't solve a problem right away, eliminating possibilities that probably don't work can give you a good idea of what possibilities might work. For example, if you forgot what you're aunt wanted you to buy at the supermarket, but you know that she's cooking lasagna for dinner, you can probably eliminate the cereal aisle, candy aisle and school supply aisle as areas in which to look.

Elimination is also a useful strategy for solving multiple choice problems. If you do not know the answer off the top of your head, try eliminating any answers you know are incorrect.

Problem 6.1: How many diagonals does a hexagon have?

a. 2
b. -4
c. 9
d. 26

Even if you don't know the formula, you can make a few good educated guesses. First, we know a hexagon isn't a square, and a square isn't a hexagon. Since a square has 2 diagonals, we know that the answer can't be (a). (b) can't be the answer because there is no such thing as a negative diagonal. Let's suppose that we're not entirely sure about what to do next, but we have a hunch that (d) is too large to be correct. Our guessing, it turns out, has payed off: the correct answer is (c).

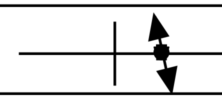
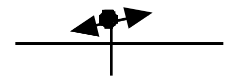
STUDY GUIDE FOR ALGEBRA 1

In algebra 1 students:

- ◇ Write and evaluate expressions
- ◇ Solve equations and inequalities
- ◇ Graph equations of lines.
- ◇ Use properties of exponents and radicals
- ◇ Manipulate polynomials

Key Vocabulary: (suggestion: quiz yourself by covering up the definitions and see if you know them)

Word	Definition	Example
Variable	A letter used to represent a number	x or y
Variable expression	A collection of numbers, variables and operations	$(2x^2 + 3x + 4)$
Exponent	The 6 in the expression 4^6	
Base	The 4 in the expression 4^6	
Equation	A statement of equality, indicated by the = sign.	$2x + 3 = 9$
Inequality	A statement of inequality, indicated by the signs <, >,≥, ≤.	$2x + 3 > 4$
Solving an equation	Finding all the numbers that make an equation true when the number is substituted in for the variable	$2x + 3 = 9$ $x = 3$
Integers	Positive and negative whole numbers including 0.	1,-3,0,-9
Distributive property	The process of distributing the number outside parenthesis to all terms inside	$2(3x + 4) = 6x + 8$

Reciprocals	Two numbers that multiply to give one.	(2/3) · (3/2) = 1
Function	A rule that establishes a relationship between two quantities.	y = 2x + 4
x- intercept	Where a graph crosses the x-axis (horizontal axis)	
y- intercept	Where a graph crosses the y-axis (vertical axis)	
Slope	The number of units a line rises vertically over the number of units it moves horizontally.	y/x or y = 1/2x + 2
Absolute Value	The distance on a number line the given value is from zero: l±xl = +x	|-3|= 3 |7|= 7

Skills Review

1. Simplifying Expressions

Find a simpler form for the expression by combining like terms.

Example: 2x + 3y – x + 4y *is simplified as* 2x-x and 3y + 4y, *which gives us* x + 7y

Simplify these:

1. 3y + 5x – 4x + 2y
2. 4(2x + 4) – 4x

2. Isolate the Variable

To isolate the variable, undo whatever is being done to the variable.

Example:

7x + 6 = 27
7x + 6 **– 6** = 27**– 6**
7x = 21
7x**/ 7** = 21**/ 7**
x = 3

Isolate the variable in the following:

1. $3x + 6 = 18$
2. $2(3x - 4) = 10$

3. Simplifying Exponents and Radicals

Find a simpler form of the expression by applying rules for exponents and radicals. First we'll take a look at rules for manipulating **exponents**:

- 2^3 means $2 \cdot 2 \cdot 2 = 8$
- $X^3 \cdot X^4 = X^7$ (when multiplying using exponents, add the exponents).
- $X^7 / X^3 = X^4$ (when dividing using exponents, subtract the exponents).
- $(X^3)^4 = X^{12}$ (when applying an exponent to a number that already has an exponent, multiply the exponents).

Solve:

1. $X^3 \cdot X^6 =$
2. $X^8 / X^3 =$
3. $(X^3)^6 =$
4. $3^4 =$

Some rules for manipulating **radicals**:

- $\sqrt{36} = 6$ because $6 \cdot 6 = 36$.
- $\sqrt{18}$ can be simplified by rewriting it as $\sqrt{9}\,\sqrt{2} = 3\sqrt{2}$
- You can only add like radicals. Example: $2\sqrt{3} + 5\sqrt{7} + 3\sqrt{3} - 2\sqrt{7} = 5\sqrt{3} + 3\sqrt{7}$
- When multiplying radical expressions you multiply numbers outside the radical sign together and numbers inside the radical sign together. For example: $2\sqrt{3} \cdot 4\sqrt{5} = 8\sqrt{15}$

Problems:

1. $\sqrt{49} =$
2. $2\sqrt{6} + 5\sqrt{11} + 3\sqrt{6} - 2\sqrt{11} =$
3. $\sqrt{24} =$
4. $2\sqrt{7} \cdot 3\sqrt{5} =$

4. Equations of Lines

The purpose of this section is to help you graph a line given an equation. To do this you will need to be knowledgeable of the meaning of slope and y- intercept

• Slope is a ratio of rise over run. On a coordinate graph that translates into change in the y-coordinates over the change in x-coordinates. In the equation $y = 2x + 4$ the slope is 2.
• The Y-intercept is where the graph crosses the y - axis. For $y = 2x + 4$ the y-intercept is 4
• An easy way to graph an equation is by placing a point at the y- intercept then tracing the slope from that point. For $y = 2x + 4$ we would place a point on the y axis at 4 and from that point move up 2 and right 1 (slope of 2 means 2/1)

Try these:

1. What is the slope and y-intercept of $y = 6x + 7$

2. What is the slope and y-intercept of $2x + 3y = 6$ (hint: put it in y = form first)

5. Polynomial Arithmetic

This section focuses on adding, subtracting, multiplying and dividing polynomials

Adding polynomials: simply remember to add like terms

• $(2x^2 + 3x + 4) + (5x^2 + 6x + 7) = (7x^2 + 9x + 11)$

Subtracting polynomials is the same as adding opposites. Multiply all of the terms of the second polynomial by -1. (In effect, you have changed the signs). Now add.

• $(2x^2 + 3x + 4) - (5x^2 -6x + 6) = (2x^2 + 3x + 4) + (-5x^2 + 6x - 6) = -3x^2 + 9x - 2$

Problems:
$(3x^2 + 3x + 4) + (9x^2 - 6x + 7)$
$(2x^2 - 3x + 4) - (5x^2 -6x + 6)$

Multiplying polynomials: two commonly used techniques are distributive property and FOIL

• Distributive prop: $2x(2x^2 + 3x + 4) = 2x\cdot 2x^2 + 2x\cdot 3x +2x \cdot 4 = 4x^3 + 6x^2 + 8x$

• FOIL stands for *First – Outside – Inside – Last* and is used on problems like (2x + 3)(2x + 3) To simplify this you multiply *First* the terms 2x • 2x; then the *Outside* terms 2x • 3; then the *Inside* terms 3 • 2x; and finally the *Last* terms 3 • 3. From this we get $4x^2$ +

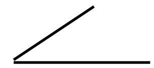
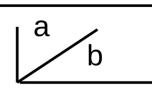
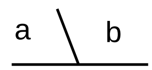
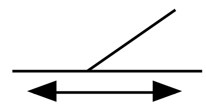
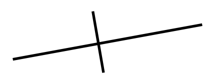
6x + 6x + 9 and then combine like terms to get $4x^2 + 12x + 9$.

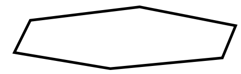
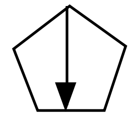
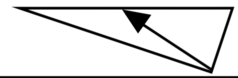
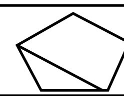
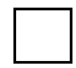
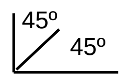
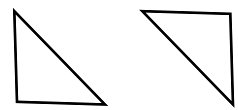
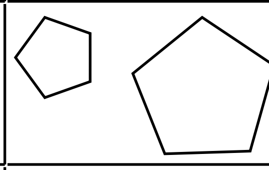
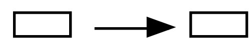
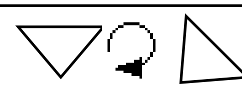
Problems:

$4x(2x^2 - 3x + 4)$
$(2x + 3) (5x + 4)$

Study Guide for Geometry

Key Vocabulary

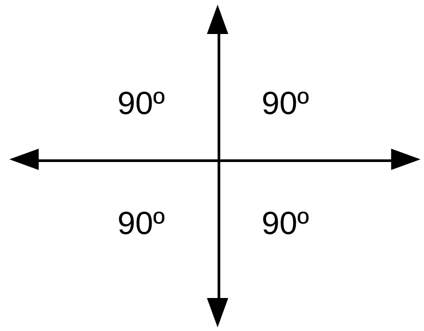
Word	Definition	Example
Congruent	Two or more line segments or angles are congruent if, when measured, they are equal.	
Right angle	An angle with measure of 90 degrees	
Acute angle	An angle with measure of less than 90 degrees	
Obtuse angle	An angle with a measure between 90 and 180 degrees	
Complementary angles	Two angles whose sum is exactly 90 degrees	
Supplementary angle	Two angles whose sum is exactly 180	
Linear Pair	Two adjacent angles that form a straight line. Linear pair are also supplementary	
Parallel Lines	Two lines in the same plane that have no points in common	
Perpendicular lines	Two lines that intersect to form a right angle	

Midpoint	The point that divides a segment into 2 parts of equal length	
Polygon	A closed figure with straight sides	
Altitude	A line segment in a polygon that is drawn from a vertex perpendicular to the opposite	
Median	A line segment in a polygon that connects a vertex to the midpoint of the opposite side	
Diagonal	A line segment in a polygon that connects a vertex to a nonadjacent vertex	
equilateral	All sides are equal length	
Angle bisector	A line segment drawn from the vertex of an angle that divides it into 2 congruent angles	
Congruent figures	Two figures that are the same size and shape. Corresponding angles are congruent and	
Similar figures	Two figures that are the same shape but a different size. Corresponding angles are congruent and corresponding sides are proportional	
Translation	Moving a geometric figure in a straight line	
Rotation	Moving a geometric figure in an angular	

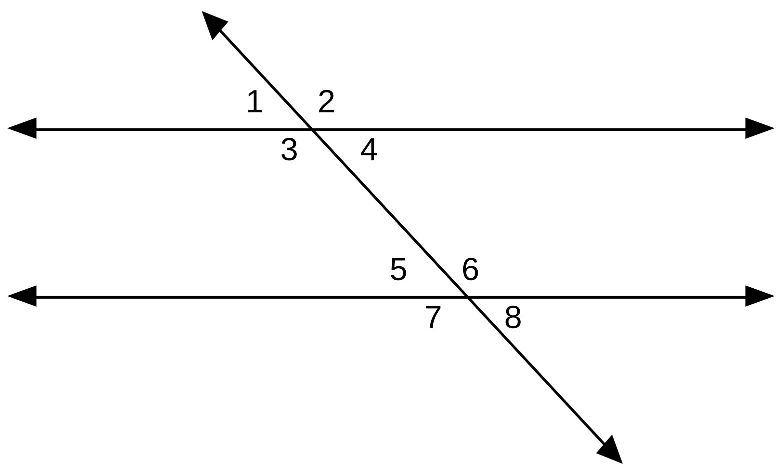
Skills Review

1. Perpendicular and Parallel Line

Perpendicular lines form right angles and have only one point in common, as in the diagram below. All 4 angles formed are 90 degrees.



Parallel lines have no points in common. Many interesting relationships develop when we draw a third line through parallel lines (this third line is called a transversal)



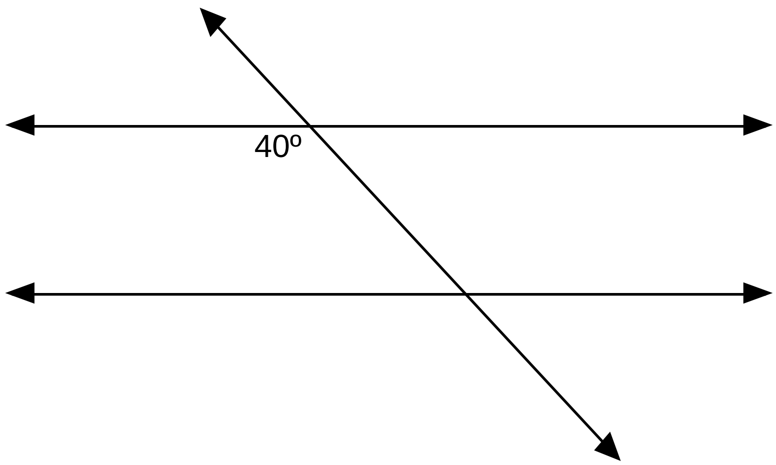
Many angle pairs are congruent. It is because of this they get special names

Congruent *alternate interior* angle pairs: 3 and 6, 4 and 5

Congruent *alternate exterior* angle pairs: 1 and 8, 2 and 7

Congruent *corresponding* angle pairs: 1 and 5, 2 and 6, 3 and 7, 4 and 8

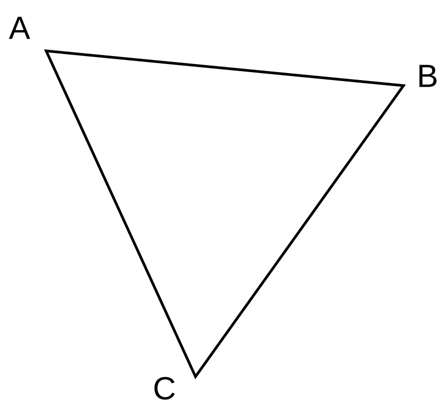
Choose any pair of angles from the figure above and they are either congruent or supplementary. With this knowledge, can you find the measure of all remaining angles in the figure below?

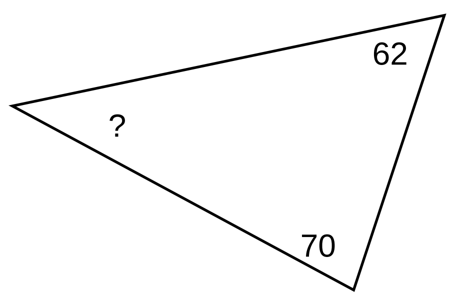


2. Triangle Relation

The purpose of this section is to investigate the many relations between angles of triangles.

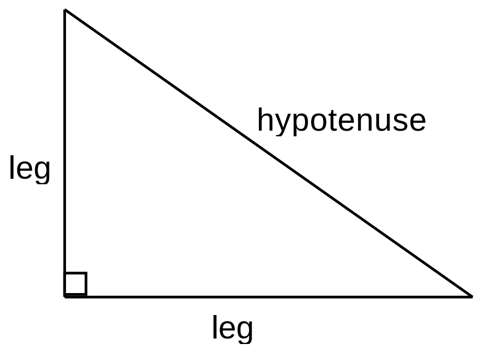
1. The first important relationship is that the sum of the interior angle of *any* triangle is 180º.

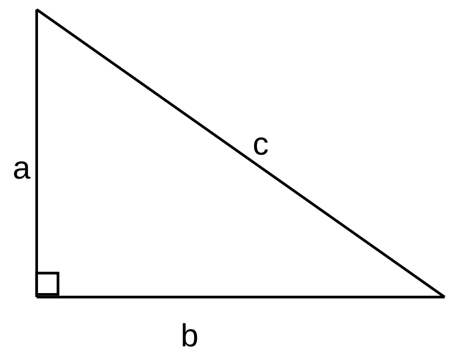




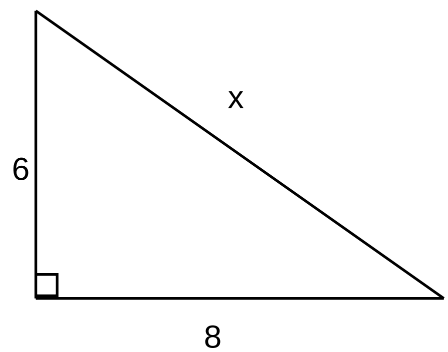
This relationship can be written as: m   A + m   B + m   C = 180. Using this formula, can you find the missing angle measurement for the triangle on the right?

2. The second important relationship is the most famous in all of geometry: the Pythagorean Theorem. It can be applied only to right triangles.



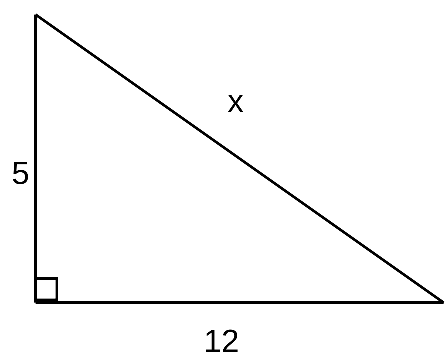


The theorem states that the square of the hypotenuse is equal to the sum of the squares of both legs, which can be written as: $c^2 = a^2 + b^2$. So, given the 2 sides of any triangle you can find the side of the third. For example,



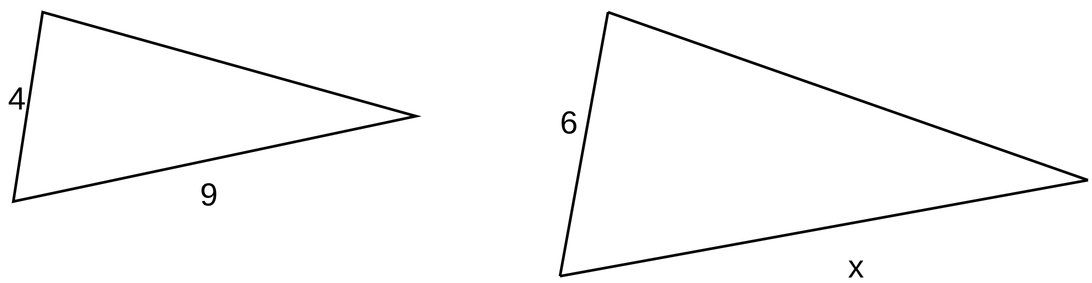
Since we know that the legs are 6 and 8, we would write out the formula for solving the problem as $x^2 = 6^2 + 8^2$. Squaring both legs, we get $x^2 = 36 + 64$. Adding the squares gives us $x^2 = 100$. All that is left is to solve for the square root of 100, written as $\sqrt{x^2} = \sqrt{100}$. So we determine that $X = 10$.

Try to solve for x in this example.



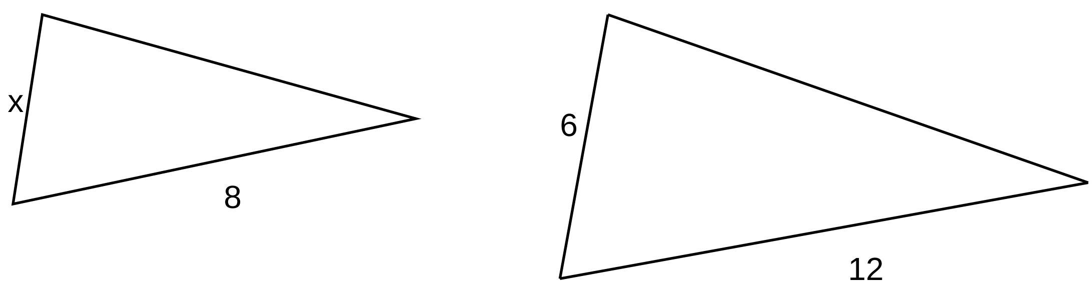
3. Another important relationship involves similar triangles. Remember that similar triangles have the same shape but are different sizes. This means that the angles are congruent and that corresponding sides are proportional. Therefore calculations made with the measurements of one triangle can help us in figuring out the measurements of a similar triangle.

Below is a set of similar triangles. We can solve for the unknown side using proportions. $4/6 = 9/x$



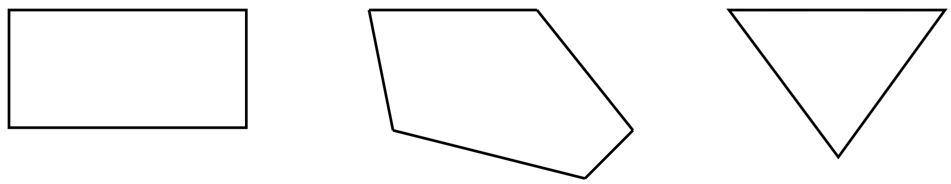
Given sides 4 and 9 for the left triangle, we can describe the relation between the two as $4/6 = 9/x$. (Can you figure out why we write $9/x$?) To figure out the ratio of difference between the triangles, we can cross multiply.

Try this figure out the value of x below:

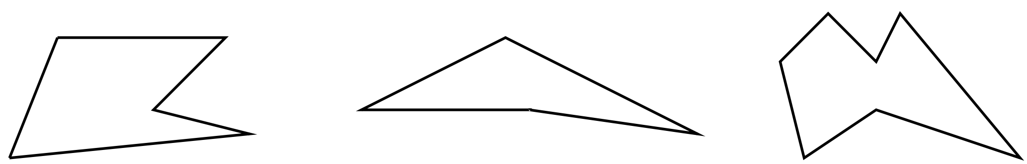


The purpose of this section is to help you to identify different types of polygons as well as recognize some of their properties. First we will look at the difference between a convex polygon and a concave polygon

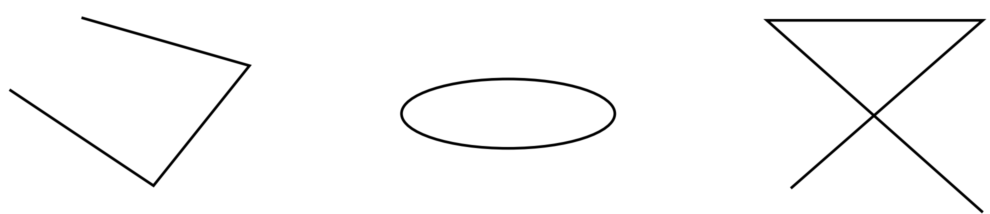
Convex Polygons



Concave Polygons



Not Polygons



Any line drawn between any vertices of a convex polygon will always be inside the polygon. For concave polygons, there is always at least one line drawn between vertices which falls outside the polygon.

3.1 Names of Polygons

Next, let's look at *polygon names*. Polygons are named according to the *number of sides*. Some of these are names are ones you've probably encountered.

Number of sides	**Name**
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Unidecagon *or* Undecagon
12	Dodecagon

Try drawing the following polygons in the boxes below:

Convex Pentagon	**Concave**

3.2 Some calculations associated with polygons.

• The sum of the interior angles of a polygon with n sides is 180(n-2)
• The measure of one interior angle of a regular polygon with n sides is (regular means all sides are congruent and all angles are congruent) $\frac{180(n-2)}{n}$
• The number of diagonals in a polygon with n sides is $\frac{n(n-3)}{2}$

3.3 Problems using the formulas above:

1) What is the sum of the interior angles of a heptagon?

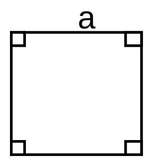
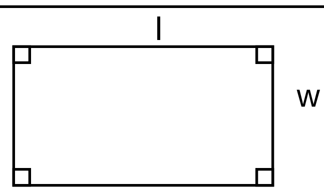
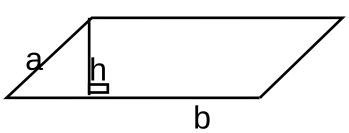
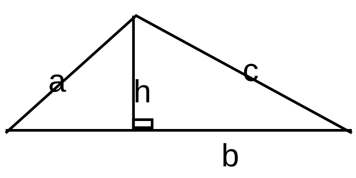
2) What is the measure of one angle of a regular pentagon?

3) How many diagonals are there in a dodecagon?

3.4 Area and Perimeter

The purpose of this section is to help you calculate the area and perimeter of convex polygons.

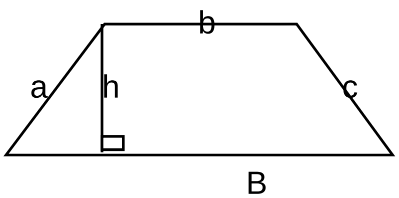
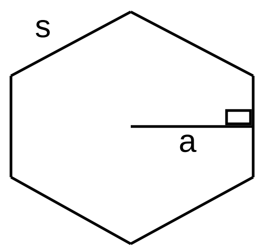
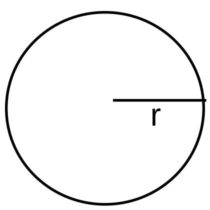
To calculate areas and perimeters it is necessary to know some formulas. These are summarized for you in the table below.

Figure	Name	Perimeter/ Circumference	Area
	square	4a	a^2
	rectangle	2l + 2w	lw
	parallelogram	2a + 2b	bh
	triangle	a + b + c	1/2 bh

3.4 Problems

1. Computer the area of a triangle with a base of 10 and a height of 5.

2. Computer the area of a trapezoid with a base of 6 and 8 and height of 10.

	trapezoid	a + b + c + B	$\frac{1}{2}(b+B)h$
	regular	6 s	1/2 a p
	circle	2 π r	π r2

Try these:

1. Compute the perimeter and area of a square with a side of 5.
2. Compute the perimeter and area of a rectangle with base 9 and height 6.
3. Compute the perimeter and area of a parallelogram with base 9 and height 6.
4. Compute the circumference and area of a circle of radius 5.

Study Guide for Algebra 2 / Trigonometry

Word	Definition	Example
A sequence	A list of numbers, each separated by a comma, that usually exhibits a pattern	2,5,8,11,14,17...
Direct variation	"y varies directly as x" means that as x gets larger, y gets larger.	y = 3 x
Inverse variation	"y varies inversely as x" means that as x gets larger, y gets smaller.	$y = \frac{3}{x}$
Relation	A set of ordered pairs	$\{(2,3),(4,6),(5,9)\}$
Domain	The set of all x-values(first number in each ordered pair) in a relation	$\{2,4,5\}$ from the above relation
Range	The set of all y-values(second number in each ordered pair) in a relation	$\{3,6,9\}$ from the above relation
Function	A relation in which none of the domain values is repeated	$\{(2,3),(4,6),(5,6)\}$
Inverse relation	A set of ordered pairs created when the ordered pairs of the original relation are reversed	$\{(3,2),(6,4),(6,5)\}$ from the above function
Quadratic equation	An equation in the following form: $ax^2 + bx + c = 0$	$ax^2 + bx + c = 0$
Radical	The bracket also known as a "square root" sign	$\sqrt{5}$
Radical equation	A equation in which the variable is under the radical sign	$\sqrt{x+4} = 7$
Trinomial	An expression containing 3 terms separated by + and - signs	$4x^3 + 9x - 7$

1. Quadratic Equations

The objective is to solve quadratic equations (find the values that make the equation true)

The First Fundamental Theorem of Algebra guarantees that any quadratic equation ($ax^2 + bx + c = 0$) has exactly 2 solutions (sometimes it is the same number but they will always have 2 solutions)

Simple quadratic equations (one term on each side of the "=" sign) can be solves by taking the square root of both sides, for example:

$$x^2 = 9$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = \pm 3$$

Notice the answers of 3 and -3. Remember there are always 2 answers

More complicated quadratic equations require more complicated mathematics. While there are several approaches that can be used, this author prefers to use the Quadratic Formula to solve the remaining types of quadratics. Although applying the quadratic formula may require some computation, you can always depend on the formula to produce the two solutions you need. Some methods which require less computation may lead to nowhere and you will need to test a different method to assure yourself of a correct answer.

The Quadratic Formula:

for $ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x^2 - 5x + 6 = 0$$
$$a = 1, b = -5, c = 6$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$
$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$
$$x = \frac{5+1}{2} or \frac{5-1}{2}$$
$$x = 3 or 2$$

Solve:

1. $x^2 + 8x + 15 = 0$
2. $x^2 + 2x - 24 = 0$

2. Radical Equations

The purpose of this section is to help you solve radical equations.

To solve a simple radical equation: you square both sides. For example, to solve the simple radical equation of $\sqrt{x} = 8$, we square both sides as follows: $(\sqrt{x})^2 = 8^2$. Solving for x gives us $x = 64$.

Be extra mindful when using squares and roots. As with most math, it's important to check your answers because sometimes getting the square root or square of a number can be more tricky than it seems. Since $\sqrt{64} = 8$, we know that our work above is correct.

To solve a more complicated radical equation, follow these steps:
1) Isolate the radical expression that involves the variable. This means to get it by itself on one side of the "=" sign.
2) Square both sides.
3) Solve the resulting equation and, as always, *check the answer.*

For example:

$$\sqrt{x+4} - 6 = 2$$
$$\sqrt{x+4} - 6 + 6 = 2 + 6$$
$$\sqrt{x+4} = 8$$
$$(\sqrt{x+4})^2 = 8^2$$
$$x + 4 = 64$$
$$x = 60$$

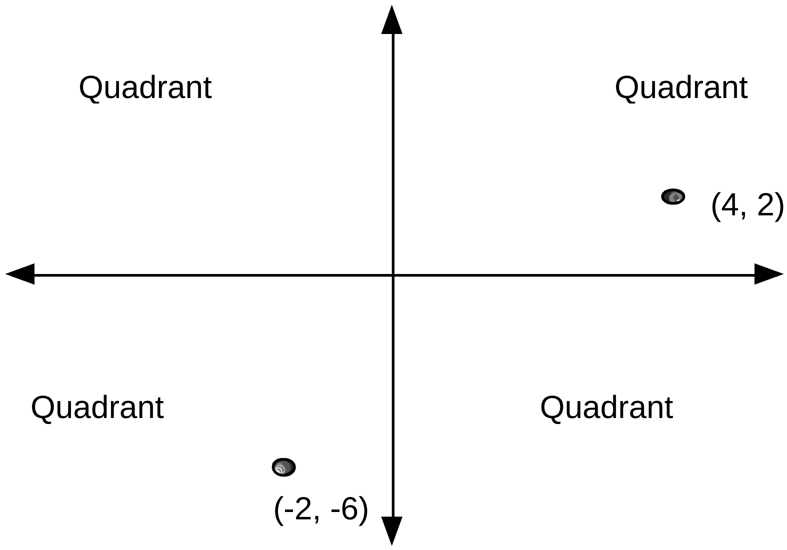
$$check: \sqrt{60+4} - 6 = 2$$
$$8 - 6 = 2$$
$$2 = 2$$

Try to solve this radical equation:

$\sqrt{x+3} - 7 = 3$

3. Problem Solving on the Coordinate Plane

The objective is to solve problems that involve ordered pairs on the coordinate graph. Although this topic is many times referred to as coordinate geometry but is a common Algebra 2 topic.



The coordinate plane is made up of 2 axes, the x-axis which is horizontal, and the y-axis which is vertical. These axis divide the plane into 4 quadrants as shown above. In these quadrants we graph points whose position is identified by ordered pairs. Two points are shown on the above graph.

Most problem solving done in the coordinate involves calculations with points. With 2 points you should be able to do three things: calculate the slope, calculate the distance between them and calculate the midpoint of a segment joining the two points. These calculations require formulas that are summarized below.

For any 2 points (x_1, y_1) and (x_2, y_2) the **slope** is the measure of steepness of a line. To find the slope, find the change in y (y_1 - y_2) divided by the change in x (x_1 - x_2).

$$\frac{x_1 - x_2}{y_1 - y_2}$$

To find the **length** or **distance** of the line segment between 2 points, imagine a right triangle and use the Pythagorean Theorem. The distance, then, between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

The **midpoint** of a segment connecting the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Try this: Find the slope, distance, and midpoint of the points (2,2) and 5,6).

4. Sequences and Series

The objective is to solve problems involving arithmetic and geometric series and sequences. It is important to know the difference between a sequence and a series. a sequence is simply an ordered list of numbers like 1, 3, 5, 7, 9... where a series is the sum of the terms of a list of numbers like 1+ 3 + 5 + 7 + 9.

4.1 Sequence

There are many different sequences. Two important sequences in algebra are **arithmetic** and **geometric**. An **arithmetic sequence** is formed by adding a number (called the common difference) to a term of the sequence to get the next term. A geometric sequence is formed by multiplying a term of the sequence by a number (called the common ratio) to obtain the next term of the sequence.

Arithmetic sequence

To find any term of an arithmetic sequence, use the formula $a_n = a_1 + (n-1)d$, where n is the number of the term you seek, a_n is the nth term, a_1 is the first term and d is the common difference.

Consider the sequence 1,3,5,7,9... If we want the 23rd term then
n=23, a_1=1 and d=2 (obtained by subtracting 2 consecutive terms.)

a_{23} = 1 + (23-1)2 a_{23} = 1+44 a_{23} =45

Geometric sequence

To find any term of an geometric sequence, use the formula
$a_n = a_1 \cdot r^{n-1}$ where n is the number of the term you seek, a_n is the nth term, a_1 is the first term and r is the common ratio.

Consider the sequence 1,3,9,27,81,... If we want the 12th term then
n=12, a_1=1 and r=3 (obtained by dividing 2 consecutive terms.)
$a_{12} = 1 \cdot 3^{12-1}$ a_{12}= 531441

Problems

1. What is the 20th term of 1,5,9,13,17...?
2. What is the 5th term of 5, 15, 45, ...?

4.2 Series

Arithmetic series

To find the sum of a specified number of terms in an arithmetic series, use the formula $S_n = \frac{n}{2}(a_1 + a_n)$ where S_n is the sum of the first n terms, a_1 is the first term,and a_n is the nth term(remember, you can calculate the nth term using the formula above)

consider the series 1+3+5+7+9... If we want to find the sum of the first 23 terms then n=23, a_1=1 and a_n=77 (as calculated above) so the sum is 23/2 (1 + 77) = 879.

Geometric series

You can find the sum of a specified number of terms in an geometric series using the formula $S_n = \frac{a_1 - a_n r}{1 - r}$ where S_n is the sum of the first n terms, a_1 is the first term, r is the common ratio and a_n is the nth term(remember, you can calculate the nth term using the formula above)

consider the series 5 + 15 + 45... If we want to find the sum of the first 5 terms then n=5, a_1=5 and a_n= 405 (as calculated above)
so the sum is $S_n = \frac{5 - 405(3)}{1 - 3} = \frac{5 - 1215}{-2} = \frac{-1210}{-2}$ = -605

Try these
1. What is the sum of the first 20 terms of 1+5+9+13+17...?
2. What is the sum of the first 8 term of 1 + 2 + 4 + 8...?